Exercises day 2, Rational homotopy theory

Exercise 1. The following is the main input in showing that the complex of polynomial differential forms gives the 'right' rational cohomology for topological spaces.

(a) Show that $\Omega^*_{PL}(\Delta^n)$ satisfies the Poincaré lemma, i.e., that the cohomology of this complex is $\mathbb{Q}$ concentrated in degree zero. (In other words, every closed form is exact.)
(b) What happens if you try to replace $\mathbb{Q}$ by a field of positive characteristic?

Exercise 2.

(a) Construct a minimal model for the space $\mathbb{C}P^n$ and compute its rational homotopy groups. (Hint: another approach to computing the rational homotopy groups of $\mathbb{C}P^n$ is via the fiber sequence

$$S^1 \to S^{2n+1} \to \mathbb{C}P^n,$$

which you can use to double-check your answer.)
(b) Construct a minimal model for the space $SU(n)$ and compute its rational homotopy groups. As in part (a), you can check your answer by contemplating the fiber sequences

$$SU(n-1) \to SU(n) \to S^{2n-1}$$

for $n \geq 2$.